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CNA

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Frederick M. Bomse

Operations Evaluation Group

CENTER FOR NAVAL ANALYSES

2000 North Beauregard Street, Alexandria, Virginia 22311

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INTRODUCTION

The investigation of digital signal processing algorithms requires the computer simulation of sensor signals having certain stochastic properties. The construction of such signals by Monte-Carlo methods generally entails the use of computer generated sequences of random numbers. Although most computer library systems have subroutines available for generating random sequences according to a variety of probability amplitude distributions, almost without exception, their elements are uncorrelated. This enables the rapid and economical simulation of combinations of primary and intefering signals (e.g., noise) in which the former is deterministic and the latter, random and uncorrelated. Alternatively both the primary and interfering signals can be represented by uncorrelated random sequences. An example would be a radar echo in which the signal of interest is random due to random fluctuations in the target cross section and the interfering signal is white Gaussian noise.

Between these two extremes is a class of interesting signals which must be represented by random, but correlated sequences. This class poses a problem for computer signal simulations because in general there is no computationally feasible way to construct a sequence of random numbers having both a specified probability amplitude distribution and a specified autocorrelation function. It is not difficult however to

generate a normally distributed sequence with a specified autocorrelation function.

BACKGROUND

The present investigation arose in connection with an attempt to simulate radar clutter echoes. Such echoes are random and usually correlated over several radar pulses. Among the detector envelope probability amplitude distributions which have been used in radar clutter modeling and analysis are the log-normal and Weibull distributions [1,2].

That clutter echoes can often be described by a log-normal distribution leads to a convenient way of simulating them. Because of the very simple transformation which relates a log-normally distributed random variable to one which is merely normally distributed, it is easy to construct log-normal sequences with specified autocorrelation functions.

This problem has in fact been addressed in the literature [3, 4]. In both of those references, the method involved the direct generation of a normally distributed sequence with a specified covariance matrix followed by an appropriate transformation. This paper adopts a slightly different approach based on a Discrete Fourier Transform of the appropriate spectral density. Since this can be implemented by means of a standard FFT routine it may be faster than the direct method.

The remainder of this paper will not deal in detail with properties of radar clutter per se, but rather with the techniques of generating correlated log-normal sequences by using correlated normal sequences. Further discussion with particular quantitative application to radar clutter modeling will be presented in a forthcoming paper.

In the following sections the relationships between the means, variances and autocorrelation functions of normal and log-normal random variables are given. The generation of correlated normal sequences using the power spectral density is discussed. The relationship between the autocorrelation functions is then invoked to produce appropriately correlated log-normal sequences. Illustrations of the method are presented in the final section. The discussion is limited to sequences which represent wide-sense stationary processes. These are processes whose means are independent of time and whose correlation functions depend only on the time interval between samples.

MATHEMATICAL TECHNIQUES

LOG-NORMAL/NORMAL RELATIONSHIPS

Let $z_1 \equiv z(t)$ and $z_2 \equiv z(t+\tau)$ be mean zero normally distributed random variables from a wide sense stationary process $\{z(t)\}$ with variance σ_z^2 and correlation coefficient $\rho = \rho(\tau)$. The probability density function of z_1, z_2 is

$$p_z(z_1 z_2) = \frac{1}{2\pi\sigma_Z^2 \sqrt{1-\rho^2}} \exp - \frac{1}{2(1-\rho^2)\sigma_z^2} \left[z_1^2 - 2\rho z_1 z_2 + z_2^2\right] \qquad (1)$$

The autocorrelation function is

$$\overline{z_1 z_2} = \overline{z(t)z(t+\tau)} = R_z(\tau) = \sigma_z^2 \rho(\tau) \qquad (2)$$

in which the over-bar denotes statistical expectation.

Upon setting $\ell n y_1 = z_1 + a$ and $\ell n y_2 = z_2 + a$ (a = constant) the variables $y_1 \equiv y(t)$ and $y_2 \equiv y(t+\tau)$ are found to be log-normally distributed with joint density function:

$$p_y(y_1, y_2) = \frac{1}{2\pi\sigma_z^2 \sqrt{1-\rho^2}\, y_1 y_2} \cdot \exp - \frac{1}{2(1-\rho^2)\sigma_z^2} \left[(\ell n y_1 - a)^2 - 2\rho(\ell n y_1 - a)(\ell n y_2 - a) + (\ell n y_2 - a)^2\right] \qquad (3)$$

By performing suitable integrations it is easy to show that y_1 and y_2 are each separately log-normal. Furthermore, the statistics of the $\{y\}$ process are simply related to those of the $\{z\}$ process as follows:

$$\bar{y}_1 = \bar{y}_2 = \bar{y} = e^{\left(a+\frac{\sigma_z^2}{2}\right)} \qquad \text{(a)}$$

$$\overline{y_1^2} = \overline{y_2^2} = \overline{y^2} = e^{2(a+\sigma_z^2)} \qquad \text{(b)}$$

$$\sigma_y^2 = \overline{y^2} - (\bar{y})^2 = e^{2a}\left[e^{2\sigma_z^2} - e^{\sigma_z^2}\right] \qquad \text{(c)}$$

$$(4)$$

$$R_y(\tau) = \sigma_y^2 \rho_y(\tau) + (\bar{y})^2 = (\bar{y})^2 e^{R_z(\tau)} \qquad \text{(d)}$$

or

$$R_z(\tau) = \frac{\ln R_y(\tau)}{(\bar{y})^2} = \ln\left[1 + \frac{\sigma_y^2}{(\bar{y})^2}\rho_y(\tau)\right] \qquad \text{(e)}$$

Equations (4, a–e) contain the key to the procedure. The relevant statistics are related by simple exponential or logarithmic transformations. For example, if $\{y\}$ is a log-normal process with

autocorrelation function $R_y(\tau)$, then under the transformation $\ell ny \to z+a$, $\{z\}$ will be a normal process with autocorrelation function $R_z(\tau)$ given by (4e).

DISCRETE FOURIER TRANSFORMS

The primary tool required for the generation of correlated sequences is the finite Discrete Fourier Transform (DFT). The properties of this transform follow logically from those of the infinite interval continuous Fourier Transform. The latter are well known. A few definitions are given here mainly to establish the notation.

Assuming the existence of all integrals, the standard Fourier transform pair $\{x(t), X(\omega)\}$ is

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega)e^{i\omega t}d\omega \qquad \text{(a)}$$

(5)

$$X(\omega) = \int_{-\infty}^{\infty} x(t)e^{-i\omega t}dt \qquad \text{(b)}$$

One important property of the pair (5a, 5b) is that for a real time function $x(t)$

$$X(-\omega) = X^*(\omega) \text{ (real } x(t)) \qquad (6)$$

where the asterisk denotes complex conjugation. If in addition, $x(t)$ is even then $X(\omega)$ is also real and even:

$$X(-\omega) = X(\omega) \quad (\text{real, even} \quad x(t)) \tag{7}$$

Given a wide sense stationary process with autocorrelation function $R(\tau)$, the spectral density $S(\omega)$ is obtained from $R(\tau)$ by a transform like (5b):

$$S(\omega) = \int_{-\infty}^{\infty} (R(\tau)e^{-i\omega\tau}d\tau \tag{8}$$

and, since $R(\tau)$ is real and even so is $S(\omega)$.

One way of effecting passage to the finite DFT is to approximate (5b) by a discrete sum over an interval $(-T/2, T/2)$. One sets $T = N\Delta t$ and $t = n\Delta t$, $n = -N/2 \ldots\ldots N/2-1$ and evaluates the resulting summation at the discrete set of frequencies $\omega_k = k\Delta\omega = \frac{2\pi k}{T}$. The result is:

$$X(k\Delta\omega) = \Delta t \sum_{n=-\frac{N}{2}}^{\frac{N}{2}-1} x(n\Delta t)e^{\frac{-i2\pi kn}{N}} \tag{9}$$

Standard practice is to shift the index n so that it runs from 0 to N-1. Setting $X(k\Delta\omega)/\Delta t = X_k$ and $x(n\Delta t) = x_n$ in (9) and performing this shift leads to the usual form of the DFT and its inverse:

$$X_k = \sum_{n=0}^{N-1} x_n e^{\frac{-2\pi ikn}{N}} \qquad \text{(a)}$$

$$\text{(10)}$$

$$x_n = \frac{1}{N} \sum_{k=0}^{N-1} X_k \, e^{\frac{2\pi ikn}{N}} \qquad \text{(b)}$$

Corresponding to (6) and (7) for real, and real and even $x(t)$ are analagous relations for x_n

$$X_k = X^*_{N-k} \quad k = 1,2 \; \ldots \; \frac{N}{2} - 1 \; (\text{real } x_n) \qquad (11)$$

$$X_k = X_{N-k} \quad k = 1,2,\ldots \; \frac{N}{2} - 1 \; (\text{real,even } x_n) \qquad (12)$$

Equations (10a) and (10b) together with the condition (12) are used extensively in the following paragraphs.

GENERATING CORRELATED SEQUENCES

To generate a correlated sequence with a specified autocorrelation function, the first step is to compute the corresponding spectral density. For this purpose equation (8) can be used provided the integration can be performed. Otherwise, an approximation using (10a) with $x_n = R_n = R(n\Delta\tau)$ will suffice. In either case, samples $S_k = S(k\Delta\omega)$ of the spectral density will be related to the given autocorrelation function by

$$R_n = \frac{1}{N} \sum_{k=o}^{N} S_k \, e^{\frac{2\pi ikn}{N}} \tag{13}$$

which is simply equation (10b) applied to the transform pair $\{R_n, S_k\}$ With these spectral density samples in hand generate two independent sequences of zero mean uncorrelated random variables $\{X_k\}$, $\{Y_k\}$ according to any arbitrary probability amplitude distribution, and form the complex random variable U_k as follows:

$$\begin{aligned} U_k &= X_k \quad k=0,\ k=\frac{N}{2} \\ U_k &= X_k + i\,Y_k \quad k = 1,2\ldots N/2-1 \end{aligned} \tag{14}$$

The X_k and Y_k must be scaled so that for each k

$$\overline{U_k U_k^*} = \overline{|U_k|^2} = 1 \tag{15}$$

Using samples of the spectral density S_k obtained from the given autocorrelation function R_n form a new set of complex variables Z_k by scaling the U_k this way:

$$Z_k = \sqrt{S_k}\, U_k \quad k=0,1\ldots N/2 \tag{16}$$

For $k = N/2+1,\ N/2+2, \ldots\ N-1$ set

$$Z_k = Z_{N-k}^* \tag{17}$$

Now perform an inverse DFT on the Z_k normalized as shown:

$$z_n = \sqrt{N}\left[\frac{1}{N}\sum_{k=0}^{N-1} Z_k e^{\frac{2\pi ikn}{N}}\right] \tag{18}$$

The sequence $\{z_n\}$ resulting from this procedure is real, and furthermore has the prescribed autocorrelation funtion R_n. That the z_n are real, follows immediately from (11) in light of condition (17).

That R_n (corresponding to S_k) is indeed the autocorrelation function of the z_n can also be easily demonstrated. For the correlation function of this sequence at a lag ℓ say is

$$\overline{z_n z_{n+\ell}} = \frac{1}{N} \sum_{k,m} \overline{Z_k Z_m^*} \, e^{\frac{2\pi kni}{N}} \, e^{\frac{-2\pi im(n+\ell)}{N}}$$

$$= \frac{1}{N} \sum_{k,m} \overline{Z_k Z_m^*} \, e^{\frac{2\pi in(k-m)}{N}} \, e^{\frac{-2\pi im\ell}{N}}$$

But $\overline{Z_k Z_m^*} = \sqrt{S_k S_m} \, \overline{U_k U_m^*} = \sqrt{S_k S_m} \, \delta_{km}$, since the U_k are uncorrelated zero mean random variables. Therefore

$$\overline{z_n z_{n+\ell}} = \frac{1}{N} \sum_{k,m} \sqrt{S_k S_m} \, \delta_{km} \, e^{\frac{2\pi in(k-m)}{N}} \, e^{\frac{-2\pi im\ell}{N}}$$

or

$$\overline{z_n z_{n+\ell}} = \frac{1}{N} \sum_{k} S_k \, e^{\frac{-2\pi ik\ell}{N}} \equiv R_\ell \tag{19}$$

In other words $\overline{z_n z_{n+\ell}}$ is identical with the inverse transform of S_k, and this is precisely R_ℓ.

The only remaining question is the probability amplitude distribution of $\{z_n\}$. Recall that $\{X_k\}$ in frequency space which spawned $\{z_n\}$ were zero mean uncorrelated sequences. But their probability distribution was not specified. The z_n however are linear combinations of the X_k and Y_k and hence the Central Limit Theorum implies that for some value N, of the size of the sequence, $\{z_n\}$ will tend toward a normal distribution. The mean of this distribution will be zero and its variance will be given by equation (19) evaluated at zero lag:

$$\sigma_z^2 = \overline{z_n^2} = \frac{1}{N} \sum_{k=0}^{N-1} S_k \tag{20}$$

An example of a sequence $\{z_n\}$ with a given correlation function, constructed from two uniformly distributed sequences $\{X_k\}$ and $\{Y_k\}$ will be given in the next section.

The technique described above for generating correlated sequences together with equation (4) relating the statistics of the log-normal and normal distributions enables the generation of correlated log-normal sequences with little difficulty. The method is summarized below.

- Given a desired log-normal process $\{y(t)\}$ with mean $\ddot{y}$, variance σ_y^2 and autocorrelation function $R_y(\tau)$, use equation (4e) to find the autocorrelation function

$R_z(\tau)$ of the underlying zero-mean normal process $\{z(t)\}$.

- Either by direct integration of equation (8) or use of the discrete transform (10a) compute samples $S_z(k)$ of the power spectral density of the $\{z(t)\}$ process.

- Apply equations (14) - (18) to obtain the sequence $\{z_n\}$. In particular, generate the sequences $\{X_k\}$ and $\{Y_k\}$ as zero mean NORMAL processes. Since a linear combination of normally distributed random variables is always itself normally distributed, this guarantees that the z_n are normally distributed zero-mean random variables.

- Set $y_n = e^{z_n+a}$ for $n = 0,1,\ldots N-1$. The constant a is determined by $\bar{y}$ and σ_y^2. According to equations (1) - (4) the y_n will be log-normally distributed with the statistics $\bar{y}$, σ_y^2 and $R_y(\tau)$. The relationships between the z and y statistics are as expressed in (4).

An example of this procedure is given in the next section.

EXAMPLES OF CORRELATED SEQUENCES

GENERAL

Two examples of the preceding methods are presented in this section. The first one, intended for comparison purposes, illustrates the generation of a random sequence with a specified autocorrelation function but with no attempt to shape the final probability amplitude distribution function. The second illustrates the generation of a log-normal sequence with this same autocorrelation. To make the comparison complete, both the mean and variance of each of the final distributions were set equal to 1.

To obtain reasonable statistical samples an ensemble of 100 sequences was generated for each example. Each of the sequences had 64 elements leading to a total of 6400 samples. The choice of sequence length is related to the specified autocorrelation function and spectral density and to the desired resolution in time and frequency, as explained below.

AUTOCORRELATION, SPECTRAL DENSITY AND RESOLUTION

A Gaussian correlation function was chosen primarily because radar clutter autocorrelation functions are often well approximated by this shape [5]. Specifically the autocorrelation function was:

$$R(\tau) = e^{-2\pi^2 \alpha^2 \tau^2} \tag{21}$$

with corresponding power spectral density $S(\omega)$ whose shape is also Gaussian:

$$S(\omega) = \frac{1}{\alpha\sqrt{2\pi}} e^{-\omega/8\pi^2 \alpha^2} \tag{22}$$

Note that $R(\tau)$ is normalized such that $R(0) = 1$.

A $\Delta\tau$ of 1 was chosen for the correlation function samples so that $\tau_n = n\Delta\tau = n$. The halfwidth of $R(\tau)$ is defined as that value of τ for which $R(\tau)/R(0) = .5$. Clearly, the choice of τ depends upon α. In both examples the value of α was chosen so that the halfwidth occurred at $\tau = 3$. (This is in rough agreement with the number of pulses over which clutter echoes are typically correlated). The resulting value of α is then determined by $\alpha^2 = \ln 2/18\pi^2$. The resolution in τ is such that four samples (at $\tau = 0,1,2,3$) fall within the halfwidth.

The halfwidth of $S(\omega)$ also depends on α, and for the value specified above it occurs at $\omega = (2\ln 2)/3$. Now the frequency bin

width $\Delta\omega$ is equal to $2\pi/N\Delta\tau = 2\pi/N$, since $\Delta\tau = 1$. To obtain about the same resolution in ω as in τ one merely sets:

$$\frac{\omega_{1/2}}{\Delta\omega} = \frac{\tau_{1/2}}{\Delta\tau}$$

where $\omega_{1/2}$, $\tau_{1/2}$ are the halfwidths of $S(\omega)$ and $R(\tau)$ and $\Delta\omega$, $\Delta\tau$ the respective bin widths. Substituting $\Delta\tau = 1$, $\Delta\omega = 2\pi/N$ and using the values of $\omega_{1/2}$ and $\tau_{1/2}$ mentioned in the preceding paragraph, a sequence length $N \simeq 41$ samples is obtained. Since the Fast Fourier Transform (FFT) algorithm used to compute the DFT operates most efficiently when N is a power of 2 , the sequence length for this analysis was chosen to be the next higher power of 2 greater than 41 or $2^5 = 64$.

Approximately 99.7% of the total power is contained within radian frequencies less than or equal to 3 standard deviations of the Gaussian in (22). This equates to an actual frequency $f = \omega/2\pi$ of .187 Hz. Twice this value or .375 Hz would be the corresponding Nyquist frequency. But the actual sampling frequency is 1 Hz so the spectrum is essentially free from aliasing.

EXAMPLE 1, SPECIFIED AUTOCORRELATION FUNCTION

This example illustrates the generation of a mean 1, variance 1 process with a Gaussian shaped correlation coefficient as expressed in

(21). The autocorrelation function differs from this coefficient only by the addition of the square of the mean value of the process. The correlation coefficient is shown in figure 1a. For this example the uncorrelated sequences $\{X_k\}$ and consequently $\{U_k\}$ were generated according to a uniform probability amplitude distribution. Samples of the spectral density given in (22) and shown in figure 1b, were used to weight the U_k . A DFT was performed on the weighted samples to obtain a correlated, mean zero time sequence. Finally, each sample was shifted by one unit to obtain a sequence with mean 1.

The sample autocorrelation function (solid curve) is compared with the desired autocorrelation function (dashed curve) in figure 2. This sample autocorrelation was computed in the following way. For each sequence k of the ensemble an estimate of the correlation function at lag ℓ was made:

$$\hat{R}_k(\ell) = \frac{1}{N-\ell} \sum_{i=0}^{N-\ell} x_i x_{i+\ell} \tag{23}$$

This estimate is unbiased. The estimates for each sequence were then averaged over the ensemble at each ℓ:

$$\bar{\hat{R}}(\ell) = \frac{1}{100} \sum_{k=1}^{100} R_k(\ell) \tag{24}$$

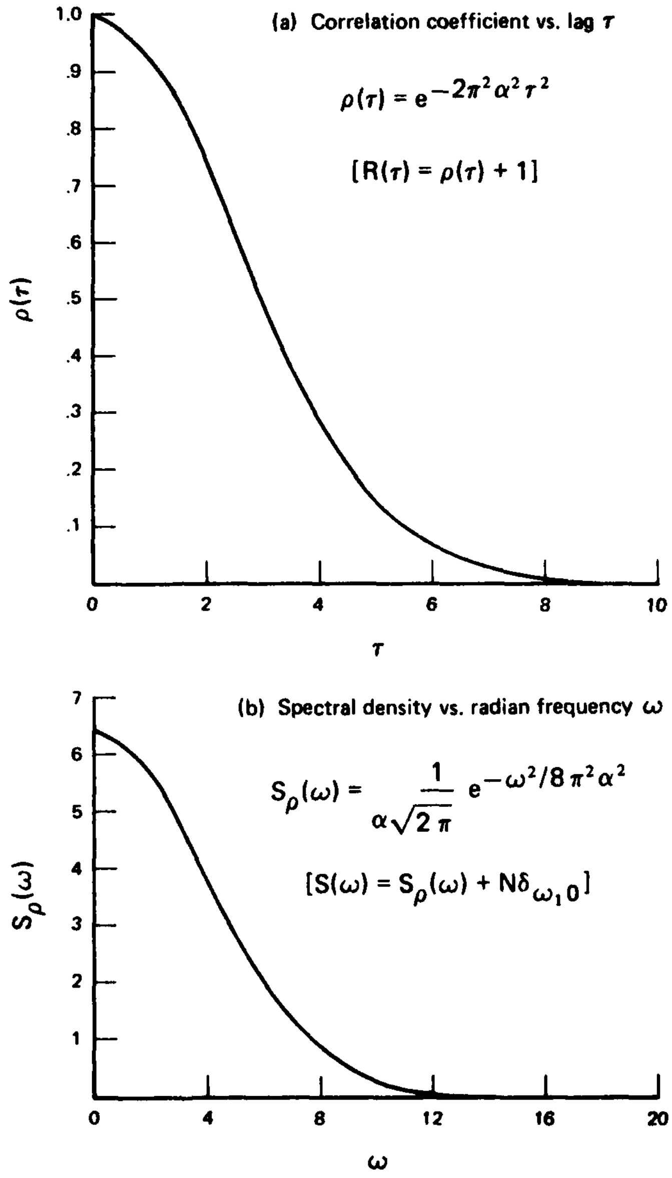

FIG. 1: SPECIFIED CORRELATION FUNCTIONS AND SPECTRAL DENSITY FOR EXAMPLE 1

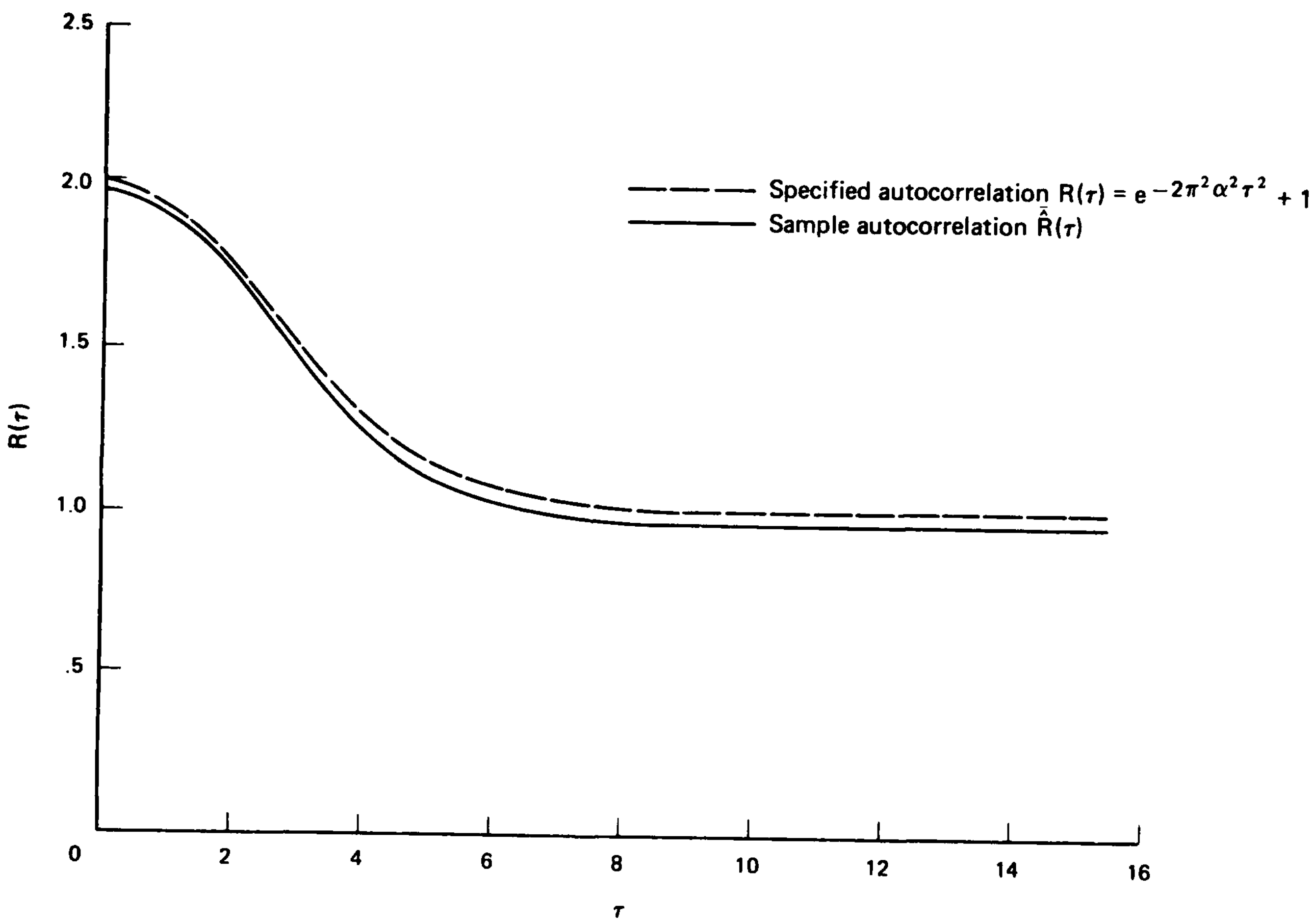

FIG. 2: SAMPLE AUTOCORRELATION FUNCTION COMPARED WITH SPECIFIED AUTOCORRELATION FUNCTION FOR EXAMPLE 1

The maximum absolute difference between these curves is about .05 or 5%.

The probability density function is shown in figure 3. The histogram is a frequency plot of the random samples and the solid curve is a theoretical normal density function with mean and standard deviation equal to 1. This theoretical curve is normalized to the area of the histogram. The left hand ordinate scale is calibrated in numbers of samples. The right scale gives $p(y)\Delta y$, the theoretical probability of finding a sample value between y and $y+\Delta y$. Here, Δy is the bin width of the histogram. Although the frequency samples were uniformly distributed, the samples in figure 3 were obtained by linear combinations of them (the DFT) and hence by the Central Limit Theorem would be expected to tend to normality. For sequences of length 64, the figure indicates that agreement with the normal distribution is excellent. However, as clutter samples these would fail in some respects. Although they are appropriately correlated as shown in figure 2, the probability density is not characteristic of clutter echoes. Not only do 16 percent of the samples have negative values, but the long positive tail often seen in clutter distributions is not present.

EXAMPLE 2, LOG-NORMAL SEQUENCE WITH SPECIFIED AUTOCORRELATION

This example illustrates the generation of a mean 1, variance 1 log-normal process with the same Gaussian shaped correlation coefficient $\rho_y(\tau)$ as used for example 1. In this case however, the

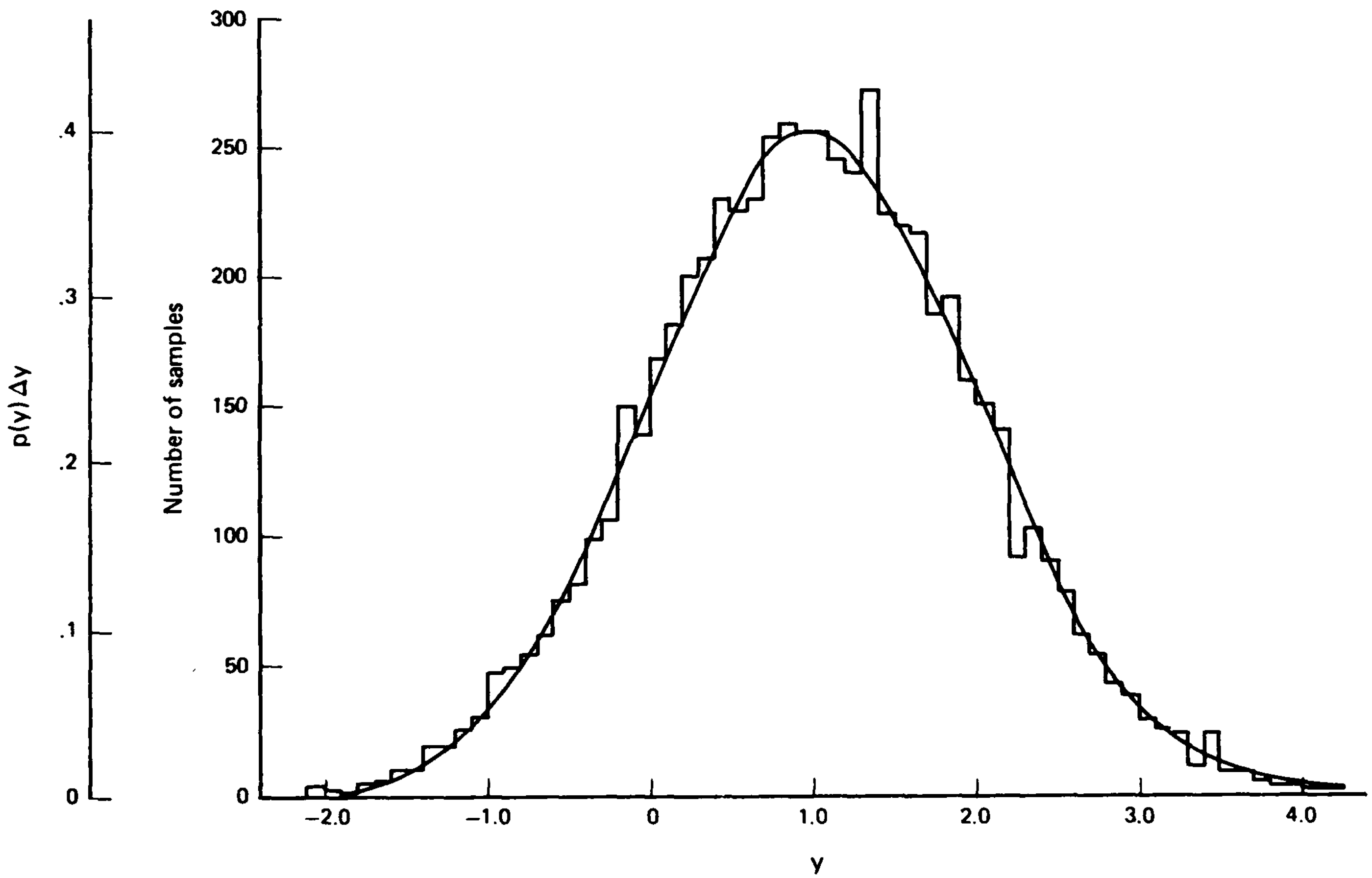

FIG. 3: PROBABILITY DENSITY FOR MEAN 1, VARIANCE 1 PROCESS OF EXAMPLE 1

spectral density of the underlying normal process must be used to weight the frequency samples. It is obtained from the autocorrelation function of that process, $R_z(\tau)$ which is related to $\rho_y(\tau)$ by equation (4e). For a mean 1, variance 1 log-normal process and the specific form of ρ_y used here (4e) becomes

$$R_z(\tau) = \ln\left[1 + e^{-2\pi^2\alpha^2\tau^2}\right] \tag{25}$$

$R_z(\tau)$ (solid curve) is compared with $\rho_y(\tau)$ (dashed curve) in figure 4a. Note that the two become equal for τ greater than about 4. This can be verified quantitatively by expanding $\ln(1+x)$ for small x.

The approximate spectral density corresponding to (25) was obtained by a DFT of that equation. It is shown in figure 4b. The independent frequency samples $\{X_k\}$, $\{Y_k\}$ and $\{U_k\}$ were generated according to a mean zero normal distribution and weighted with appropriate values of this spectral density. A DFT of the weighted samples resulted in a mean zero normally distributed time sequence whose autocorrelation function was the $R_z(\tau)$ of equation (25). The transformation

$$y = e^{(z+1)} \tag{26}$$

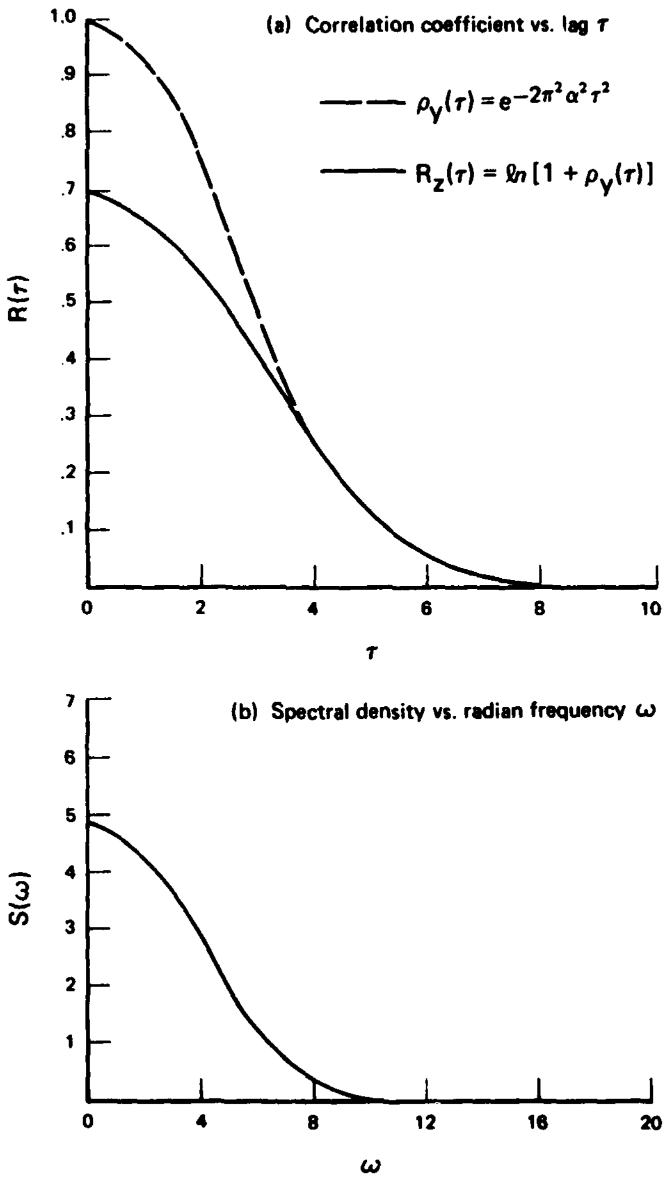

FIG. 4: SPECIFIED CORRELATION FUNCTIONS AND SPECTRAL DENSITY FOR EXAMPLE 2

applied to this sequence resulted in the desired log-normal sequence with mean 1, variance 1 and autocorrelation function $R_y(\tau) = \rho_y(\tau)+1$.

The sample autocorrelation function (solid curves) is compared with the specified autocorrelation (dashed curve) in figure 5. Two ensembles of 100 sequences each, and labeled A and B in the figure, are shown, to illustrate the statistical variability. In this case, the maximum deviation from the specified function is about 13 percent. The probability distributions of each of these ensembles is presented in figures 6 and 7. As before the histograms are frequency plots of the random samples. The solid curves are theoretical log-normal density functions with mean and variance both equal to 1. The scales on the left and right hand ordinates are exactly as described in figure 3. The agreement between the histograms and theoretical curves is excellent in both cases.

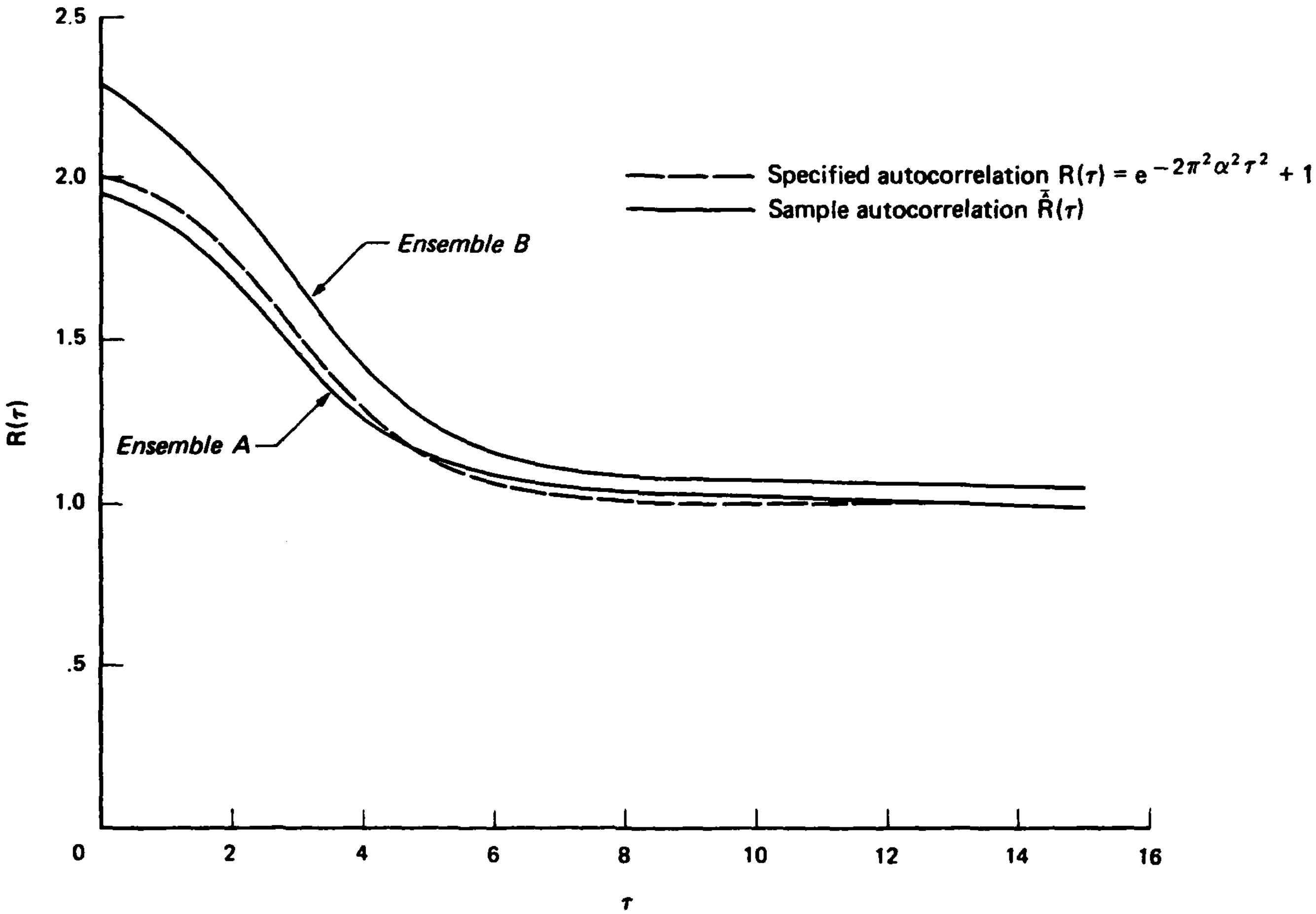

FIG. 5: SAMPLE AUTOCORRELATION FUNCTIONS COMPARED WITH SPECIFIED AUTO CORRELATION FUNCTION FOR EXAMPLE 2

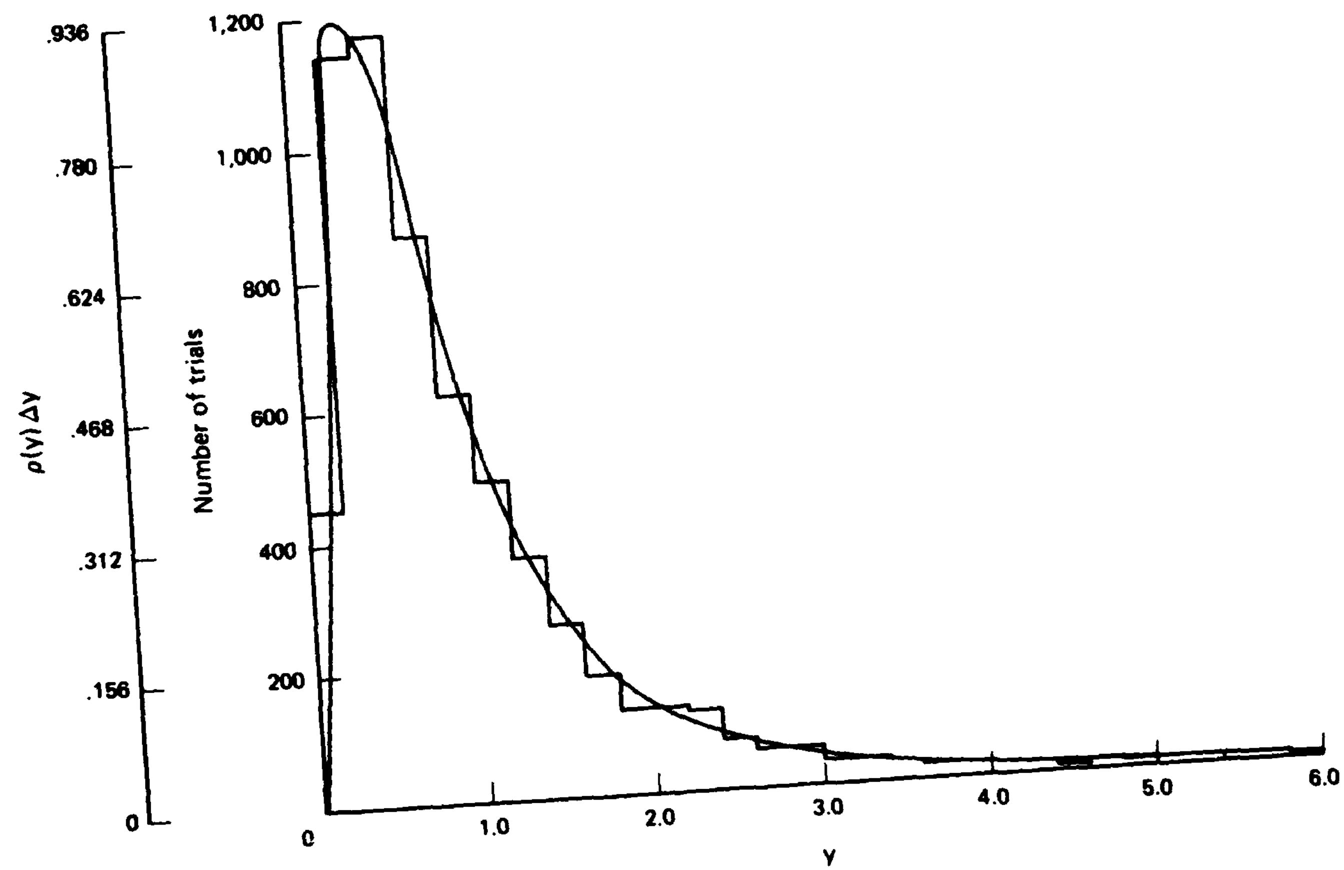

FIG. 6: PROBABILITY DENSITY FOR MEAN 1, VARIANCE 1
LOG-NORMAL PROCESS
(SAMPLE A)

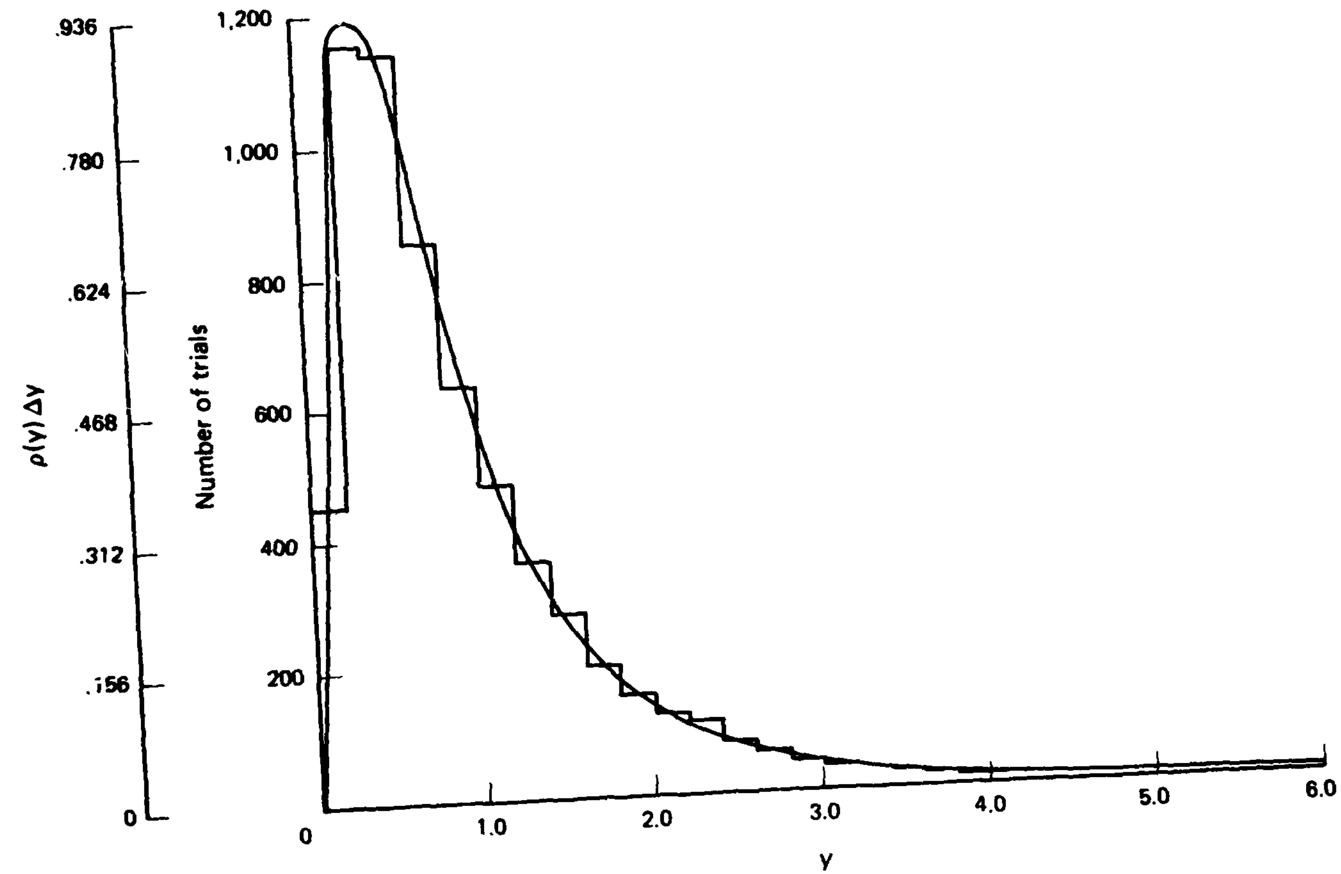

FIG. 7: PROBABILITY DENSITY FOR MEAN 1, VARIANCE 1
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(SAMPLE B)

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